Enrollment No: ____________________ **Exam Seat No:**____________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name:Complex Analysis

Subject Code:5SC01COA1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 25/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Is $f(z) = z + \bar{z}$ real valued function? (01)

b. State C- R equation in polar form. (01)

c. State De Moivre's theorem. (01)

d. Define Entire function and give one example of it. (02)

e. Prove that $\sin(ix) = i \sin hx$. (02)

Q-2 **Attempt all questions** **(14)**

a. Suppose $f(z) = u + iv, z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$ (06)

b. Prove that composition of continuous function is continuous. (04)

c. Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos(\frac{n\pi}{3})$. (04)

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove C – R equation for an analytic function. (06)

b. If $f(z) = \begin{cases} \frac{ax^3-by^3}{ax^2+by^2} + i\frac{ax^3+by^3}{ax^2+by^2} & , z \neq 0 \\ 0 & , \quad z = 0 \end{cases}$ then prove that C-R equations are satisfied at origin. (04)

c. Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic. Find harmonic conjugate of $u(x,y)$. (04)

Q-3		**Attempt all questions**	**(14)**
	a.	If $f(z)$ is regular function of z then prove that $$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\,\lvert f'(z)\rvert^2$$	(05)
	b.	Evaluate $\int_c \frac{dz}{z^2+9}$ where c is (a) $\lvert z-3i\rvert = 4$ (b) $\lvert z\rvert = 5$.	(05)
	c.	Evaluate: $\int_c \frac{e^z}{z(z-1)^3}\,dz$ where c: $\lvert z\rvert = 2$.	(04)

OR

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Cauchy's Theorem.	(07)
	b.	State and prove chain rule for derivatives.	(07)

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: (i) Pole (ii) Removable singularity.	(02)
	b.	Which are the fixed points of $w=\frac{5z-4}{5+z}$?	(02)
	c.	State Maximum modulus principal.	(01)
	d.	Write Maclaurin's series of $\cos z$.	(01)
	e.	Give an example of removable singularity.	(01)

Q-5		**Attempt all questions**	**(14)**
	a.	State and prove fundamental theorem of algebra.	(06)
	b.	Evaluate $\oint_C \frac{z}{(z-2)^2(z-1)}dz; c: \lvert z-2\rvert = 0.5$ by using Cauchy's residue theorem.	(04)
	c.	Expand $f(z) = \frac{1}{z}$ as a Taylor's series about the point $z = 1$.	(04)

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Integrate the function $f(z)=(\bar{z})^2$ from 0 to $2+i$ path is from $(0,0)$ to $(2,0)$ along the real axis and then from $(2,0)$ to $(2,1)$.	(06)
	b.	Find bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0\ \infty$.	(04)
	c.	Expand Laurent's series $\frac{1}{z(z-1)^2}$ at the point $z = 1$.	(04)

Q-6		**Attempt all questions**	**(14)**
	a.	State and prove Cauchy's inequality and deduce Liouville's theorem.	(07)
	b.	State and prove Taylor's theorem.	(07)

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove residue theorem.	(07)
	b.	Evaluate: $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx$	(07)